PROCESSING OF 3D IxJxT TABLES. THE CASE OF TRADE BETWEEN GREECE AND COUNTRIES FROM EIGHT GEOGRAPHICAL AREAS OF THE PLANET 2002-2004

Introduction

The application of Correspondence Analysis to ordinary contingency tables has proven without a doubt very effective and useful. In this study we will present the capability of the method to process more complex tables, specifically tables that include values of three variables. Let be that we have data in T={t|T=1,…k} contingency tables that we wish to study simultaneously depicted on the same factor level, which would have derived from the method of Correspondence Analysis.

Initially, we symbolize with I,J,T the totals of the levels of the three variables.

In this case the data can be presented in two ways, either in the form of a rectangular parallelepiped whose dimension of base would be equal to IxJ and its height shall consist of T levels (Figure 1a) or as a sequence of dual input tables IxJ. (Figure 1b)

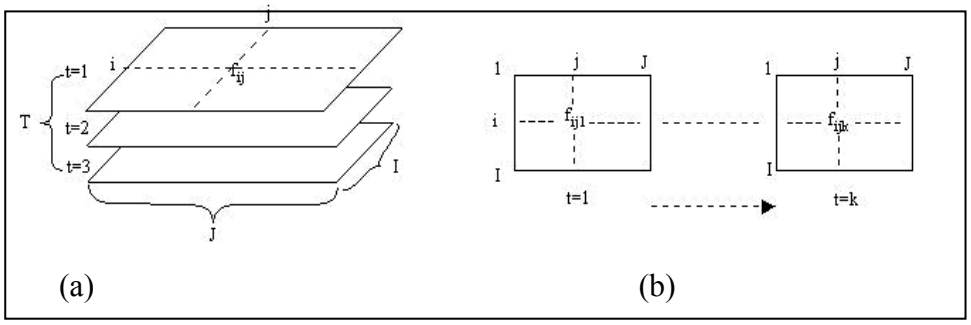

Figure 1: Graph of 3D data

The generalization of the Correspondence Analysis which would replace the «dual» analysis with a «triadic», was impossible because it was not feasible to realize the symmetric role of three clouds N(I), N(J) and N(T), and their view on the same factor plane, as is the case with «dual» analysis due to the types of transition (i) and (ii), which allow the simultaneous viewing of the clouds N(I) and N(J) on various factorial levels.

Where

$$F_a(i) = \frac{1}{\sqrt{\lambda_\alpha}} \sum_{j=1}^{p} \frac{f_{ij}}{f_i} G_a(j) \qquad (i)$$

$F_A(i)$ the coordinate of «Object-line» of the data table

L_a the characteristic root of a factor axis with a=1,2,…,p-1

F_{ij} the relative frequency of k_{ij} observation of the data table

F_I the marginal distribution of the i line

G_A the coordinate of the j variable of the data table

$$G_a(j) = \frac{1}{\sqrt{\lambda_\alpha}} \sum_{t=1}^{\nu} \frac{f_{ij}}{f_j} F_a(i) \qquad (ii)$$

Where f_j the marginal distribution of the j variable of the data table

Of course we must not forget that real problems are **never** conditionally symmetric under the effect of three variables. Usually 3D tables shall be regarded as a sequence k of 2D tables, who verify the same variables with the aim of comparing the information contained in the k tables. The proposed technique by J.P. Benzecri provides for the creation of two tables called **augmented as to J** and the second **augmented as to I**.

Initially, a 2D table M(I,J) is formed by the sum of the T tables IxJ as to variable J, who has the general datum $m_{ij} = \sum_{t=1}^{k} m(i,j,t)$. Then the T tables IxJ are placed next to it creating the augmented table as to J, while the augmented table as to I is created if the T tables IxJ are placed one under the other as shown in figure 2.

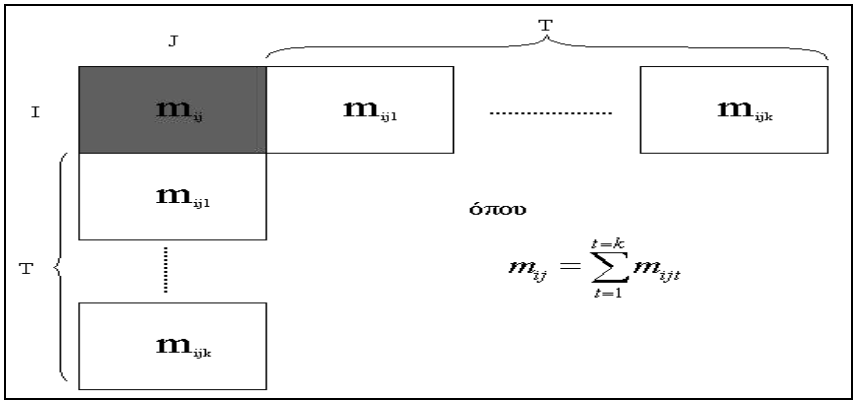

Figure 2: Graph of augmented tables as to J and as to I

This technique uses the capability of Correspondence Analysis to utilize numerous additional data without causing deterioration in the structure of the data to be displayed in factor levels, having previously calculated the coordinates of the factor axes in accordance with relations (iii) and (iv).

So Table M with general datum m_{ij} is analyzed with the use of Correspondence Analysis while the data of the T tables IxJ are regarded as additional information.

In the case of the augmented table as to J we use the following relationship to find the S supplementary data (columns)

$$G_a(s) = \frac{1}{\sqrt{\lambda_a}} \sum_{i=1}^{n} \frac{f_{is}}{f_s} \cdot F_a(i) \qquad \text{(iii)}$$

While in the case of the augmented table as to I we use the following relationship to find the S supplementary data (lines)

$$F_a(s) = \frac{1}{\sqrt{\lambda_a}} \sum_{j=1}^{p} \frac{f_{sj}}{f_s} \cdot G_a(i) \qquad \text{(iv)}$$

With the creation and then the analysis of the augmented tables we achieve the simultaneous presentation of the levels of the three variables I, J, T on the factor levels from where we derive the appropriate conclusions.

Presentation of the data

The data that will be used are educational, they do reflect, however, the true trends of the phenomenon being studied.

Table 1 of the data is therefore 3D as it examines simultaneously three characteristics of a) the trade of Greece (imports and exports) in million euros b) the areas where these transactions take place c) the time period for which we have these results.

The countries were divided in eight areas. The European Union of 15 (EU-15), North America (B_AM), the other OECD countries (Y_OOS), Center & East Europe (K.A_E.U.), N. Africa & Middle East (BA_MA), South east Asia (NAASI), Latin America (LAT_A) and all other countries (LOIPE).

In essence we create two crosstab tables. The first relates to imports of Greece from the eight areas over the period 2002-2004, while the second its exports to the eight areas over the same period.

Table 1: Trade of Greece in million Euro						
	Imports in mill €			Imports in mill €		
Areas	**2002**	**2003**	**2004**	**2002**	**2003**	**2004**
EU-15	14560	16028	17462	4708	4319	4281
North America	920	1276	1635	730	624	516
Other OECD	2559	2392	2196	524	558	602
Center & East Europe	3592	3454	3319	1980	1998	2008
N. Africa & Middle East	1790	2237	2765	425	483	560
South East Asia	2240	2037	1819	165	154	141
Latin America	158	222	291	41	37	32
Other countries	1282	1600	1913	360	310	264
TOTAL	**27166**	**29325**	**31562**	**9389**	**8483**	**8404**

Statistical analysis

For the statistical treatment of the data of the example we applied Correspondence Analysis. The analyses were performed using the MAD software (Karapistolis D. 2002), while basic source for the presentation of the above methods was the book «Multivariate Statistical Anlysis» (Karapistolis D. 2011). Both studies of Papadimitrou C (1991) and Papadimitrou - Florou (1992) are of interest and concern the same object of study.

Initially, we create the augmented table as to J, which is analyze with Correspondence Analysis based on which is given the capability to study the development of the trade balance of Greece for the period 2002-2004, as it is formed with the other countries that were grouped in eight areas.

The 1st factor explains 75.33% of total inertia, the 2nd factor 22.75%, while the sum of the two factors interprets 98.07% of the total information of the data table.

Table 2: Coordinates, views and contributions of imports and exports of the period 2002-2004 on the 1st and 2nd factor axis

	#G1	COR	CTR	#G2	COR	CTR
2002	5	5	0	75	975	199
2003	0	34	0	2	446	0
2004	-6	6	0	-72	969	194
20021	-97	436	73	109	561	314
20031	-95	998	75	2	0	0
20041	-93	478	77	-97	520	279
20022	316	954	258	-28	7	6
20032	328	998	263	1	0	0
20042	316	951	245	21	4	3

The labels 2002, 2003, 2004 concern the total of imports and exports, while the labels 20021, 20031, 20041 concern the import figures for the years 2002, 2003, 2004, while the labels 20022, 20023, 20042 concern the respective exports, data obtained in the analysis as additional «objects»

Observing the factor plane 1x2 we conclude the following:

The 1st factor axis compares the imports (20021,20031,20041) the period 2002-2004 with exports of the same period (20022,20032,20042) with the emphasis on exports and the downward trend they present from the 1the quadrant ($F_1>0$, $F_2>0$) toward the 4the quadrant ($F_1>0$, $F_2<0$) of the factor plane.

The 2th factor axis illustrates the upward trend in imports with trend from the 2nd ($F_1<0$, $F_2>0$) to the 3rd ($F_1<0$, $F_2<0$) quadrant.

By calculating the trade balance of Greece with each one of the eight areas of the planet, we conclude that it is **deficient** in all cases**.**

The special feature is that the evolution of the trade balance of the areas identified on the 1st and 2the quadrant presents a declining trend, while in 3 and 4the quadrant an upward trend, for different reasons in each quadrant, due to different development rates of imports in relation to their respective exports.

Table 3: Coordinates, views and contributions of the regions on the 1st and 2the factor axis

	#F1	COR	CTR	#F2	COR	CTR
EU-15	-18	691	11	-12	265	14
N_AM	169	673	94	-108	270	126
Y_OOS	-62	267	18	100	718	168
KA_EU	234	932	520	62	65	120
BA_MA	-86	332	34	-110	550	189
NAASI	-264	774	262	140	220	247
LAT_A	-148	440	8	-167	558	36
LOIPE	-110	602	39	-87	381	82

The selection of «objects» and «variables» that contribute to the creation of factor axes while simultaneously confer the characteristics with which the specialist interprets the axes, is performed using two criteria.

(a) they must have viewing quality greater than 200

$$COR \geq 200$$

(b) they must contribute to the creation of the axes with a percentage generally resulting from the ratio:

$$CTR = \frac{1000}{9} \approx 110$$

In accordance with the two previous conditions, the points chosen for these four factor axes per quadrant are as follows:

In the 1the quadrant ($F_1>0$, $F_2>0$) is detected the area where the deficit of the trade balance presents a declining trend as the rate of imports gradually decreasing as the rate of exports increases, which is encouraging for the Greek economy. Such is the area of the countries of Central and Eastern Europe ($COR_1=932$, $CTR_1=520$).

In the 2the quadrant ($F_1<0$, $F_2>0$) is detected the region of South East Asia ($COR_1=774$ $CTR_1=262$) where the deficit of the trade balance is declining, as the decrease in the rate of exports is lower than the decrease of imports.

In the 3the quadrant ($F_1<0$, $F_2>0$) is detected the region of N. Africa and Middle East ($COR_2=550$ $CTR_2=189$) where the deficit of the trade balance presents an upward (discouraging element), because the rate of the increase in imports is higher than the rate of the increase in exports.

In the 4the quadrant ($F_1<0$, $F_2>0$) is detected the region of N. America ($COR_2=270$ $CTR_2=126$) where the deficit of the trade balance appears upward, as the increase in the rate of imports is simultaneous with a decrease in exports, a situation totally negative for the Greek economy.

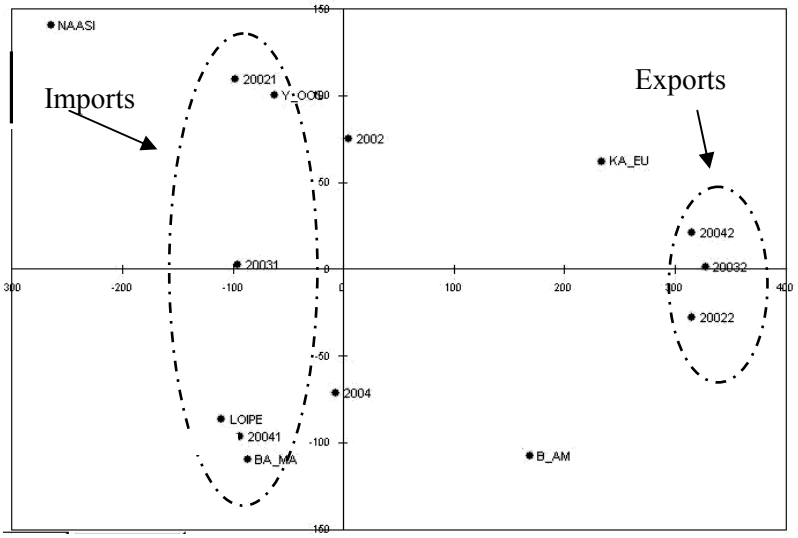

Figure 3: Factor level 1x2 of the augmented table as to J

The factor plane of figure 3 is an overview of the following data

Table 4

Trade balance Deficit=(import-export)				**Progress**	**Progress rate Imports (%)**		**Progress rate Exports (%)**	
B_AM	2002	2003	2004		2003/2002	2004/2003	2003/2002	2004/2003
KA_EU	190	652	1119	Upward	38.26	28.13	-14.52	-17.3
BA_MA	1612	1456	1311	Downward	-3.84	-3.91	0.91	0.50
NAASI	1365	1754	2205	Upward	24.97	23.60	13.65	15.94
	2075	1883	1678	Downward	-9.06	-10.70	-6.67	-8.44

Following, by creating the augmented table as to there is the opportunity to study the trade balance of Greece for the period 2002-2004 with each area individually.

The analysis of the augmented table as to I with Correspondence Analysis presents the following findings.

The 1th factor axis interprets 99.56% of total inertia.The 2th the remainder 0.44%. Due to this reason, in the graph it is shown only the distribution of the table data on the 1st factor axis.

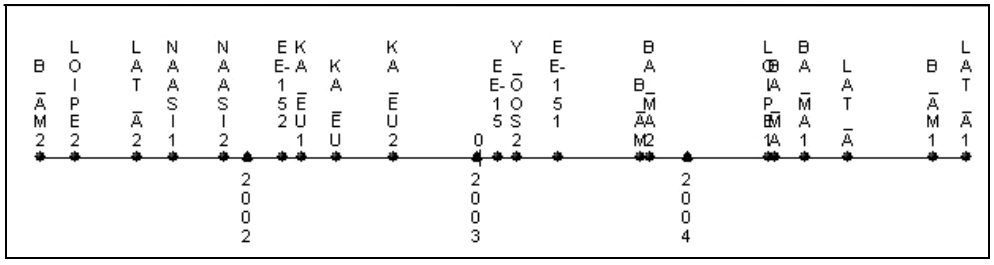

Figure 4: The 1th factor axis of the augmented table as to I

A more graphic illustration of the areas N.America and EU_15 is given in the figure below.

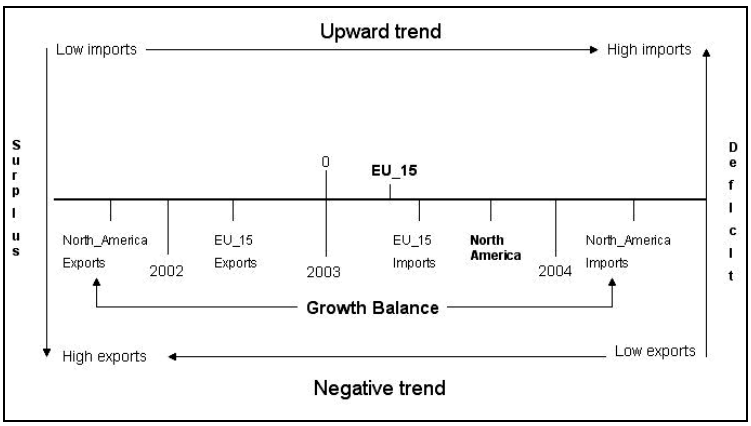

Figure 5: Diagram of the trade balance of Greece with the regions North _America and EU_15

The study of figure initially shows that the balance of trade with the countries of America and the countries of the European Union of 15, is in deficit in so far as the trend of imports into Greece from these areas is higher than the export trend of Greece in the same areas over the period 2002-2004. With a second reading we see that the deficit with the countries of the North America is greater than that which our country presents with the countries of EU of 15. Therefore, the 1th factor axis presents a quantitative comparative scale of the deficit of the trade balance of Greece with the eight areas to which were arranged the countries of the world.

Conclusion

The proposed technique for processing tables which verify values of three variables is absolutely successful because it enables the analyst of similar data to depict on a factor level simultaneously the three variables and conclude with the same ease with which he would by studying simple contingency tables. In the case where we examine the longitudinal progress of data of a 2D contingency table then we can see on the factor level the **dynamics** of the phenomenon, which arises from the synthesis of the data, in contract with the classic factor level which depicts on the factor axes just the deviations of «objects» I (lines of the data table) and the «variables» J (columns of the data table) from the barycenters of the clouds N(I) and N(J) respectively.

So, in the example presented while the data concerned to the imports and exports done by Greece in the period 2002-2004 with the countries of eight areas of the world, on the factor level of the augmented table as to J, there was depicted the rate of progress of the deficit of the trade balance of our country with these areas.

Obviously the original data in Table 1 are completely different from those presented in Table 2 and illustrated on the factorial plane 1x2.